Applications of the Duckworth-Lewis Method

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Abstract

The Duckworth-Lewis method is steadily becoming the standard approach for resetting targets in interrupted one-day cricket matches. In this note we show that the Duckworth-Lewis resource table can be used to quantify the magnitude of a victory in one-day matches. This simple and direct application is particularly useful in breaking ties in tournament standings and in quantifying team strength.

Keywords: one-day cricket matches, estimation.

1. Introduction

There are 4 possible outcomes in one-day cricket matches:

(1) the team batting first can win in a non-abandoned match

(2) the match can end in a tie

(3) the match can be abandoned

(4) the team batting second can win in a non-abandoned match

In the first case, the run differential between the 2 teams is a sensible measure of the magnitude of victory. In the second case, which is rare in practice, the tie itself indicates that there is zero magnitude of victory. In the third case, either the game is declared null or one of the teams is declared the winner. In the latter event, a projected score is determined for the team batting second, and again, magnitude of victory can be assessed by calculating the run differential. However, in the fourth case, the magnitude of victory is unclear because the match terminates as soon as the team batting second scores more runs than the team batting first even though the team batting second may have leftover wickets and overs.

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Why should we care about quantifying the magnitude of victory? Without such quantification, statistical analyses are typically based on binary data corresponding solely to wins and losses. For example, de Silva and Swartz (1997) estimate the effect of the home team advantage in one-day international cricket matches using logistic regression. It is a generally accepted statistical principle that data is valuable and that one should not "waste" data by needless summarisation. Therefore, by quantifying the magnitude of victory in one-day cricket matches, future statistical modelling can better utilise the information contained in matches. Quantifying the magnitude of victory may aid in assessing team strength, determining betting strategies, breaking ties in tournament standings, etc.

How can we quantify the magnitude of victory? The Duckworth-Lewis method (Duckworth and Lewis, 1996, 1998a, 1998b) is steadily becoming the standard approach for resetting targets in interrupted one-day cricket matches. At the time of writing, the Duckworth-Lewis method has been adopted for various competitions by the Zimbabwe Cricket Union, the England and Wales Cricket Board, New Zealand Cricket, and most notably, the International Cricket Council. In Section 2, we show that a simple application of the Duckworth-Lewis resource table can be used to quantify the magnitude of victory in one-day cricket matches. In Section 3, we provide an example whereby a three-way tie could have sensibly been broken using our approach. In Section 4, a second example is given which models the strength of ICC (International Cricket Council) nations in one-day international cricket matches.

2. Quantifying the Magnitude of Victory

The Duckworth-Lewis resource table (see Table 1) was devised to improve "fairness" in interrupted one-day matches. The resource table is based on the principle that resources are diminished in a shortened match and that targets should be reset according to the resources available. Duckworth and Lewis (1996, 1998a) obtained the entries in the resource table using statistical methods based on historical match data. For a brief introduction to the Duckworth-Lewis method, see the CricInfo website (http://www.cricket.org).

Our problem, as stated in Section 1, is to quantify the magnitude of victory when the team batting second wins in a non-abandoned match. Using the Duckworth-Lewis resource table, this is a straightforward task. We determine the resource percentage used U relative to a standard 50-over match. We then solve $UE/100 = R$ for E where E is the number of effective runs and R is the actual number of runs scored. Here, the actual number of runs is simply the proportion of the effective number of runs in accordance with the resources used. With effective runs, an effective run differential can then be calculated to quantify the magnitude of victory.

Consider then the most complicated scenario possible where the batting team starts and stops batting n times. When they start batting on the i-th occasion they have a_i resources available according to Table 1, and similarly, when they stop batting they have b_i resources available. Therefore, the team has used $U = \sum_{i=1}^{n}(a_i - b_i)$ resources relative to a standard 50-over match. For example, consider a 50-over match where 10 overs are played and 1 wicket is lost. A rain delay occurs and the inning is shortened to 20 remaining overs which are then played out. In this case, $a_1 = 100.0$, $b_1 = 84.5$, $a_2 = 56.7$ and $b_2 = 0.0$.

The simplicity of the approach is part of its appeal. Further, we note that in any shortened match, we may want to scale the number of runs as described above so that they are comparable to a standard 50-over match.

3. An Example Concerning Ties

We refer to the Asia Cup played in Sharjah, UAE in April 1995. This tournament featured India, Sri Lanka, Pakistan and Bangladesh in 50-over matches with the top 2 teams advancing to the championship final. After the initial round, Bangladesh had 0 wins and 3 losses. The remaining teams each had 2 wins and 1 loss. Under the three-way tie, India and Sri Lanka advanced to the finals based on superior run rates.

Now it is widely accepted that run rates can be unfair. Suppose then that we had used the methods of Section 2 to break the three-way tie. The relevant details of the initial round matches involving India, Sri Lanka and Pakistan are given in Table 2. In match 1, Pakistan has a differential of 97 runs over India. In match 2, India had lost only 2 wickets in 33 overs plus 1 out of 6 balls when it exceeded Sri Lanka's run total. Therefore, in a 50-over match, India had 16.83 of its overs left. Interpolating from Table 1, India had 48.17% of its resources remaining. Therefore, we calculate India's effective runs E by solving $(100 - 48.17)E/100 = 206$. This gives India 397 effective runs and an effective run differential of $397 - 202 = 195$ over Sri Lanka. A similar calculation in match 3 gives Sri Lanka an effective run differential of 118 over Pakistan.

Putting these results together, India has $195 - 97 = 98$ net runs, Sri Lanka has $118 - 195 = -77$ net runs and Pakistan has $97 - 118 = -21$ net runs. Therefore, our approach would have advanced India and Pakistan to the championship final rather than India and Sri Lanka. We note that the same conclusion is reached when the matches involving Bangladesh are included in the calculation.

4. An Example which Models Team Strength

The data used in this analysis are the results of full 50-over one-day international matches involving the 9 nations of the ICC. We consider all matches in the 1990's up to and including the final of the 1999 World Cup of Cricket, held in England. There are 623 such matches, the results of which are available from the "Archive" link at the CricInfo website.

As in Section 3, an effective run differential is calculated for every match. For matches in which the team batting first wins, the effective run differential is simply the actual run differential. For matches in which the team batting second wins, the effective run differential is calculated using the actual runs of the team batting first and the effective runs of the team batting second obtained via the Duckworth-Lewis resource table. We let indices $i, j = 1, \ldots, 9$

correspond to the 9 ICC nations and let $k = 0, \ldots, 9$ correspond to the site of a match where $k = 0$ refers to a neutral site. We then consider the model

$$d_{ijk} = \tau_i - \tau_j + \gamma_{ijk} + \epsilon_{ijk} \tag{1}$$

where the response variable d_{ijk} is the effective run differential (i.e. team i minus team j) for a match at site k, τ_i is a measure of strength for the i-th team such that $\sum_{i=1}^{9} \tau_i = 0$, γ_{ijk} is the home field advantage such that

$$\gamma_{ijk} = \begin{cases} \gamma & \text{if } k = i \\ -\gamma & \text{if } k = j \\ 0 & \text{otherwise} \end{cases}$$

and the ϵ_{ijk} are independent and identically distributed Normal$(0, \sigma^2)$ errors. This 10-parameter model is taken over all 623 matches. It is sensible in that the determination of team strength takes into account not only victories and losses, but also the magnitude of the victories and losses, the strength of the opponent and the site of the match.

To give more emphasis to recent matches, we consider a weighted least squares approach where the weight 1 is assigned to every match in 1999, the weight 9/10 is assigned to every match in 1998, the weight 8/10 is assigned to every match in 1997, etc. In S-plus, the function glm (Venables and Ripley, 1994) is used to estimate the model parameters. The results are given in Table 3. We observe an ordering for the ICC nations where South Africa is the strongest team and Zimbabwe is the weakest. These estimated parameters can be used to forecast the outcomes of matches. For example, should South Africa play Zimbabwe in Johannesburg, we would expect South Africa to win by $27.4 - (-37.6) + 12.5 = 77.5$ effective runs. We also observe that the home field advantage $\gamma = 12.5$ is meaningful in one-day international cricket matches. Note that the error $\sigma = 52.4$ is somewhat large; this highlights the variability in cricket matches, where unlike sports such as rugby, a considerably weaker team has a realistic chance at upsetting a stronger team.

The results in Table 3 are particularly useful for betting purposes. Consider a match between Australia and New Zealand in Auckland. Using model (1) and a continuity correction, the estimated probability of a win by Australia is

$$\begin{aligned} \text{Prob}(d_{144} > .5) &\approx \text{Prob}(17.1 - (-14.3) - 12.5 + \epsilon > .5) \\ &\approx \text{Prob}(Z > -18.4/52.4) \\ &\approx .64 \end{aligned}$$

where Z is a standard normal random variable.

We remark that different weightings have been considered with little effect on the results. Also, we have considered different parameterizations for the home field advantage. For example, some very popular teams (e.g. the West Indies) may actually experience a home team advantage at a neutral site. Again, such changes do not greatly effect the results. Various residual plots indicate that the fitted model is adequate. More extensive discussion of the model and the analysis is found in Pond (1999).

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Table 1: Abbreviated version of the Duckworth-Lewis resource table. The table entries indicate the percentage of resources remaining in a match with the specified number of wickets lost and overs available.

	Wickets Lost									
Overs Left	0	1	2	3	4	5	6	7	8	9
50	100.0	92.4	83.8	73.8	62.4	49.5	37.6	26.5	16.4	7.6
40	90.3	84.5	77.6	69.4	59.8	48.3	37.3	26.4	16.4	7.6
30	77.1	73.1	68.2	62.3	54.9	45.7	36.2	26.2	16.4	7.6
20	58.9	56.7	54.0	50.6	46.1	40.0	33.2	25.2	16.3	7.6
19	56.8	54.8	52.2	49.0	44.8	39.1	32.7	24.9	16.2	7.6
17	52.3	50.6	48.5	45.8	42.2	37.2	31.5	24.4	16.1	7.6
16	49.9	48.4	46.5	44.0	40.7	36.1	30.8	24.1	16.1	7.6
10	34.1	33.4	32.5	31.4	29.8	27.5	24.6	20.6	14.9	7.5
5	18.4	18.2	17.9	17.6	17.1	16.4	15.5	14.0	11.5	7.0
1	3.9	3.9	3.9	3.9	3.9	3.8	3.8	3.7	3.5	3.1
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 2: Summary of the relevant initial round matches involving India, Sri Lanka and Pakistan in the 1995 Asia Cup as described in Section 3. We let Team 1 denote the team batting first and Team 2 denote the team batting second. An asterisk denotes the winner of a match.

Date	Team 1 (Runs/Wickets/Overs)	Team 2 (Runs/Wickets/Overs)
Apr 07	Pak (266/9/50)*	Ind (169/10/42.4)
Apr 09	SL (202/9/50)	Ind (206/2/33.1)*
Apr 11	Pak (178/9/50)	SL (180/5/30.5)*

Table 3: Summary of the data and the estimated parameters as described in Section 4.

Parameter	Wins (Matches)	Est (Std Err)
Australia (τ_1)	114 (184)	17.1 (4.4)
England (τ_2)	31 (74)	-2.1 (6.8)
India (τ_3)	78 (166)	-3.7 (4.6)
New Zealand (τ_4)	54 (142)	-14.3 (5.2)
Pakistan (τ_5)	104 (195)	5.5 (4.3)
South Africa (τ_6)	90 (137)	27.4 (5.0)
Sri Lanka (τ_7)	67 (149)	-4.0 (4.9)
West Indies (τ_8)	57 (116)	11.7 (5.9)
Zimbabwe (τ_9)	18 (83)	-37.6 (6.2)
home field (γ)		12.5 (3.5)
standard deviation (σ)		52.4